



Subject: Grade 12 Mathematics

Topic: Mathematics: Term 1 - Mini-Exam

Total: 48 Marks

1. A: 3 475

4 marks

Explanation:

To determine the value of the series, the number of terms to be added together needs to be determined. The position of the last term in the series will be the number of terms in the series.

143 is the value of the last term. Use a T_n formula for an arithmetic sequence to determine the position of the $T_n = 143$.

$$\begin{aligned}T_n &= a + (n - 1)d \\143 &= -4 + (n - 1)(3) \\147 &= 3n - 3 \\150 &= 3n \\n &= 50\end{aligned}$$

There are 50 terms in the series. Determine S_{50} for an arithmetic series.

$$\begin{aligned}S_n &= \frac{n}{2}[2a + (n - 1)d] \\&= \frac{50}{2}[2(-4) + (50 - 1)(3)] \\&= 25[-8 + 147] \\&= 3\,475\end{aligned}$$

2. B: $y = 2^x$

4 marks

Explanation:

Determine the equation of f by substituting $(4; 2)$ into the function.

$$\begin{aligned}y &= \log_a x \\2 &= \log_a 4 \quad \text{Convert to exponential form} \\a^2 &= 4 \\a &= 2\end{aligned}$$

$$\therefore f(x) = \log_2 x$$

To determine f^{-1} , swap the x and y -values:

$$\begin{aligned}x &= \log_2 y \quad \text{Convert to exponential form} \\2^x &= y\end{aligned}$$

$$\therefore f^{-1} = y = 2^x$$



3. false

2 marks

Explanation:

$$\begin{aligned} & \frac{\sin 4\theta - \sin 2\theta}{\cos 4\theta + \cos 2\theta} && \text{Change } 4\theta \text{ to a double angle} \\ &= \frac{\sin 2(2\theta) - \sin 2\theta}{\cos 2(2\theta) + \cos 2\theta} && \text{Expand double angle} \\ &= \frac{2 \sin 2\theta \cos 2\theta - \sin 2\theta}{2 \cos^2 2\theta - 1 + \cos 2\theta} && \text{Factorise} \\ &= \frac{\sin 2\theta(2 \cos 2\theta - 1)}{(2 \cos 2\theta - 1)(\cos 2\theta + 1)} && \text{Cancel out} \\ &= \frac{\sin 2\theta}{\cos 2\theta + 1} && \text{Expand double angle} \\ &= \frac{2 \sin \theta \cos \theta}{\cos^2 \theta - \sin^2 \theta + \sin^2 \theta + \cos^2 \theta} && \text{Use square identity for 1} \\ &= \frac{2 \sin \theta \cos \theta}{2 \cos^2 \theta} && \text{Simplify} \\ &= \frac{\sin \theta}{\cos \theta} \\ &= \tan \theta \end{aligned}$$

4. true

2 marks

Explanation:

$f^{-1}(x)$ will be the inverse of $f(x)$. To determine the equation of an inverse function, the x and y -values are swapped.

$$f(x) = \frac{x - 1}{3}$$

To determine inverse:

$$x = \frac{y - 1}{3} \quad \text{Rearrange to the form } y = \dots$$

$$\begin{aligned} 3x &= y - 1 \\ y &= 3x + 1 \end{aligned}$$

$$\therefore f^{-1}(x) = 3x + 1$$

5. false

2 marks

Explanation:

For a geometric series to converge, $-1 < r < 1$.

Determine $r = \frac{T_2}{T_1}$

$$r = \frac{4(x-1)^2}{2(x-1)}$$
$$= 2(x-1)$$

Solve for x :

$$-1 < 2(x-1) < 1 \quad \text{Divide by 2}$$

$$-\frac{1}{2} < x-1 < \frac{1}{2} \quad \text{Add 1}$$

$$\frac{1}{2} < x < \frac{3}{2}$$

6. false

2 marks

Explanation:

$$f(x) = -\log_2 x$$

To change the sign of a logarithmic function, invert the base.

$$f(x) = \log_{\frac{1}{2}} x$$

To convert a logarithmic function to an exponential function, the inverse is applied (swap x and y values).

$$x = \log_{\frac{1}{2}} y$$

$$\left(\frac{1}{2}\right)^x = y$$

Therefore from f to g , there is first a reflection over the line $y = x$.

$$g(x) = \left(\frac{1}{2}\right)^x + 1$$

Then the function is shifted one unit up.

The transformation from f to g is a reflection over the line $y = x$ and a vertical translation 1 unit up.

7. A: $-\sin^2 \theta$

4 marks

Explanation:

$$\frac{\sin(-180^\circ - \theta) \cdot \tan(-\theta) \cdot \sin(90^\circ - \theta)}{\sin^2(180^\circ + \theta) + \cos^2 \theta}$$

Note: To simplify $\sin(-180^\circ - \theta)$ we use the concept of the negative angle in quadrant IV.

$$\begin{aligned}\sin(-180^\circ - \theta) &= \sin(-(180^\circ + \theta)) \\ &= -\sin(180^\circ + \theta) \\ &= -(-\sin \theta) \\ &= \sin \theta\end{aligned}$$

$$\therefore \frac{\sin \theta \cdot (-\tan \theta) \cdot \cos \theta}{(-\sin \theta)^2 + \cos^2 \theta} \quad \text{Apply quotient identity}$$

$$= \frac{\sin \theta \cdot \left(-\frac{\sin \theta}{\cos \theta}\right) \cdot \cos \theta}{(\sin^2 \theta) + \cos^2 \theta} \quad \text{Simplify}$$

$$= \frac{\sin \theta (-\sin \theta)}{1} \quad \text{Square identity in denominator}$$

$$= -\sin^2 \theta$$

Explanation:

This is a quadratic sequence because there is a constant second difference. A quadratic sequence is in the form $T_n = an^2 + bn + c$.

Sequence: $5; 2; -3; -10; \dots$

First difference: $-3; -5; -7; \dots$

Second difference: $-2; -2; \dots$

To determine the general term for a quadratic sequence:

$$2a = \text{constant second difference}$$

$$2a = -2$$

$$a = -1$$

$$3a + b = \text{first difference}$$

$$3(-1) + b = -3$$

$$b = 0$$

$$a + b + c = \text{first term}$$

$$-1 + 0 + c = 5$$

$$c = 6$$

$$\therefore T_n = -n^2 + 6$$

$$T_{30} = -(30)^2 + 6$$

$$= -894$$

9. C: -2

4 marks

Explanation:

$$\sin^2 2A + \cos^2 2A = 1$$

$$\therefore \sin^2 2A = 1 - \cos^2 2A$$

$\cos(90^\circ + A)$ is a co-function. $90^\circ + A$ is in quadrant 2 where $\cos A$ is negative, so the co-function remains negative.

$$\cos(90^\circ + A) = -\sin A$$

$$\frac{\sqrt{1 - \cos^2 2A}}{\cos(-A) \cdot \cos(90^\circ + A)}$$

$$= \frac{\sqrt{\sin^2 2A}}{\cos A \cdot -\sin A} \quad \text{Simplify}$$

$$= \frac{\sin 2A}{-\sin A \cdot \cos A} \quad \text{Expand double angle}$$

$$= \frac{2 \sin A \cos A}{-\sin A \cos A} \quad \text{Cancel out}$$

$$= -2$$

10. true

2 marks

Explanation:

$$\frac{\sin(-2x)}{\cos(180^\circ - x)} \quad \text{Reduce according to quadrant}$$

$$= \frac{-\sin 2x}{-\cos x} \quad \text{Expand double angle}$$

$$= \frac{-2 \sin x \cos x}{-\cos x} \quad \text{Simplify}$$

$$= 2 \sin x$$

11. false

2 marks

Explanation:

$$13 \cos B + 12 = 0$$

$$\cos B = -\frac{12}{13}$$

$\cos B < 0$ in quadrant 2 and 3.

$\sin B > 0$ in quadrant 1 and 2.

$\therefore B$ is in quadrant 2.

$$x = -12$$

$$r = 13$$

$$\therefore x^2 + y^2 = r^2$$

$$y^2 = 13^2 - (-12)^2$$

$$y^2 = 25$$

$$y = 5$$

In quadrant 2 the y -value will be positive.

$$\sin 2B = 2 \sin B \cos B$$

$$= 2\left(\frac{y}{r}\right)\left(\frac{x}{r}\right)$$

$$= 2\left(\frac{5}{13}\right)\left(\frac{-12}{13}\right)$$

$$= -\frac{120}{169}$$

12. true

2 marks

Explanation:

Inverse functions are reflections of each other over the line $y = x$. In other words, the x and y values swap.

$$A(1 ; 3) \text{ and } A'(3 ; 1)$$

$$B(0 ; 1) \text{ and } B'(1 ; 0)$$

The points are reflections over the line $y = x$, and the graphs, are therefore inverses of each other.

13. B: $\theta = 35,8^\circ + k.90^\circ; k \in \mathbb{Z}$

4 marks

Explanation:

When a trigonometric equation has $\sin \theta$ and $\cos \theta$, or the same angle attached to $\sin$ and $\cos$ such as $\sin 2\theta$ and $\cos 2\theta$, then the quotient identity can be applied:

$$\frac{\sin \theta}{\cos \theta} = \tan \theta$$

$$\sin 2\theta - 3 \cos 2\theta = 0 \quad \text{Isolate the functions}$$

$$\sin 2\theta = 3 \cos 2\theta \quad \text{Divide by } \cos 2\theta$$

$$\frac{\sin 2\theta}{\cos 2\theta} = \frac{3 \cos 2\theta}{\cos 2\theta} \quad \text{Simplify}$$

$$\tan 2\theta = 3 \quad \text{Determine general solution}$$

The general solution for $\tan \theta = c$ is $\theta = \tan^{-1}(c) + k.180^\circ; k \in \mathbb{Z}$

$$2\theta = 71,565...^\circ + k.180^\circ \quad \text{Divide by 2}$$

$$\theta = 35,8^\circ + k.90^\circ; k \in \mathbb{Z}$$

14. D: $\sum_{n=0}^6 3(2)^n$

4 marks

Explanation:

$\sum$ means to add. The left of the sigma is the general term for the terms in the series.

$3; 6; 12; \dots$ is a geometric series with a constant ratio of 2.

$$\begin{aligned} T_n &= a.r^{n-1} \\ &= 3(2)^{n-1} \end{aligned}$$

The first term is 3. There are 2 options:

$$3(2)^{1-1} \text{ if } n = 1 \quad \text{or} \quad 3(2)^0 \text{ if } n = 0$$

Therefore $3(2)^n$ is also an option for the general term if the first substitution of n is $n = 0$.

$$3(2)^6 = 192. \text{ The last substitution for } n \text{ is } n = 6.$$

$$\sum_{n=0}^6 3(2)^n$$

15. true

2 marks

Explanation:

Determine the ratio by calculating $\frac{T_2}{T_1}$.

$$r = \frac{1}{6} \div \frac{1}{2} = \frac{1}{3}$$

For a sum to infinity, $-1 < r < 1$, therefore $r = \frac{1}{3}$ fits into the criteria.

$$\begin{aligned} S_{\infty} &= \frac{a}{1-r} \\ &= \frac{\frac{1}{2}}{1-\frac{1}{3}} \\ &= \frac{3}{4} \end{aligned}$$

16. A: $BC = h\sqrt{25 - 24\cos^2 x}$

4 marks

Explanation:

If we can determine a measurement for AB , then the cosine rule can be used in $\triangle ABC$.

$\triangle ABD$ is a right-angled triangle. Therefore the angle of depression of 30° is the same as the angle of elevation of 30° inside $\triangle ABD$ at B .

In $\triangle ABD$:

$$\sin 30^\circ = \frac{h}{AB}$$

$$AB = \frac{h}{\sin 30^\circ}$$

$$= \frac{h}{\frac{1}{2}}$$

$$= 2h$$

In $\triangle ABC$:

$$BC^2 = AB^2 + AC^2 - 2(AB)(AC) \cos \hat{BAC}$$

$$= (2h)^2 + (3h)^2 - 2(2h)(3h) \cos 2x$$

$$= 4h^2 + 9h^2 - 12h^2 \cos 2x$$

$$= 13h^2 - 12h^2 \cos 2x \quad \text{Take } h^2 \text{ out as a common factor}$$

$$= h^2(13 - 12 \cos 2x)$$

$$BC = h\sqrt{13 - 12 \cos 2x} \quad \cos 2x = 2 \cos^2 x - 1$$

$$= h\sqrt{13 - 12(2 \cos^2 x - 1)}$$

$$= h\sqrt{13 - 24 \cos^2 x + 12}$$

$$= h\sqrt{25 - 24 \cos^2 x}$$

Total: 48 Marks